

Is gravitino still a warm dark matter candidate?

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ABSTRACT: We make use of the phase space density approach to discuss gravitino as a warm dark matter candidate. Barring fine tuning between the reheat temperature in the Universe and superparticle masses, we find that *warm* gravitinos have both appropriate total mass density, $\Omega_{\tilde{G}} = \Omega_{\text{DM}} \simeq 0.2$, and suitable primordial phase space density at low momenta provided that their mass is in the range $1 \text{ keV} \lesssim m_{\tilde{G}} \lesssim 15 \text{ keV}$, the reheat temperature in the Universe is low, $T_{\text{R}} \lesssim 10 \text{ TeV}$, and masses of some of the superparticles are sufficiently small, $M \lesssim 350 \text{ GeV}$. The latter property implies that the gravitino warm dark matter scenario will be either ruled out or supported by the LHC experiments.

KEYWORDS: Cosmology of Theories beyond the SM, Supersymmetric Standard Model.

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1. Introduction and summary

The predictions of the Λ CDM model are in outstanding consistency with the bulk of cosmological observations [1] (see also ref. [2] and references therein). Yet there are clouds above the collisionless cold dark matter scenario, which have to do with cosmic structure at subgalactic scales. Three most notable of them are missing satellites [3], cuspy galactic density profiles [4] and too low angular momenta of spiral galaxies [5]. All these suggest that CDM may be too cold, i.e. that the vanishing primordial velocity dispersion of dark matter particles may be problematic. Hence, one is naturally lead to consider warm dark matter (WDM) scenarios [6–9].

There are several ways to describe the difference between WDM and CDM scenarios. The most prominent one is that warm particles filter primordial power spectrum on small scales, and thus the formation of small halos is suppressed. The filtering scale must be small enough, since the power spectrum shows no significant deviations from the CDM prediction on scales within reach of current observations. This leads to constraints on the primordial velocity dispersion of WDM particles [10]. On the other hand, in order to improve on structure formation, the filtering scale must be of the order of the scale of missing satellites, which is believed to be of order $10^7 - 10^8 M_\odot$ [11].

We have calculated linear matter power spectrum in Λ WDM cosmology assuming that dark matter particles have the Fermi-Dirac primordial distribution function, normalized to correct present total density:

$$f(p) = \frac{\rho_{\text{DM}}}{6\pi \zeta(3) m T_{0,\text{eff}}^3} \frac{1}{e^{p/T_{0,\text{eff}}} + 1}, \tag{1.1}$$

where m is the WDM particle mass, $T_{0,\text{eff}} \equiv T_0 \left(\frac{g_{*0}}{g_*}\right)^{1/3}$; g_* and $g_{*0} \equiv \frac{43}{11}$ are the effective number of relativistic degrees of freedom at the epoch of dark matter particle production and at present epoch, respectively. To this end we have modified the Boltzmann evolution equations implemented in the Code for Anisotropies in the Microwave Background

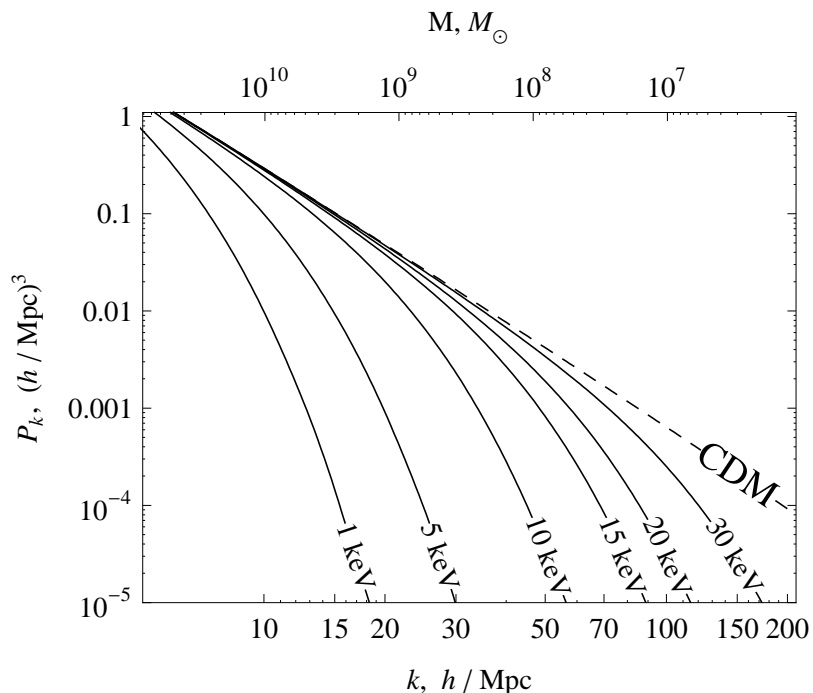


Figure 1: Linear matter power spectrum for standard Λ CDM cosmology (dashed line) and Λ WDM (solid lines) assuming the distribution of WDM particles as given by (1.1) with masses $m = 1, 5, 10, 15, 20$ and 30 keV and $g_* = g_{\text{MSSM}}$.

(CAMB) [12]. Figure 1 presents the resulting Λ WDM power spectrum for $m = 1, 5, 10, 15, 20$ and 30 keV (solid) in comparison with Λ CDM (dashed). In the WDM case, g_* is chosen to be equal to $g_{\text{MSSM}} = 228.75$, the maximum number of relativistic degrees of freedom equilibrated in plasma in the framework of MSSM. One concludes that the power spectrum is suppressed by about an order of magnitude on the scales corresponding to $10^8 M_\odot$ and smaller provided the WDM particle mass is smaller than about 15 keV. Thus we consider a particle as a WDM candidate if its mass obeys

$$m \lesssim 15 \text{ keV} . \tag{1.2}$$

Of course, this is an indicative figure, not a strict upper limit.

Another way to quantify the notion of warm dark matter is to make use of the phase space density approach [13–15]. Its key ingredient is the ratio between the mass density and the cube of the one-dimensional velocity dispersion in a given volume, $Q \equiv \rho/\sigma^3$. On the one hand, this quantity is measurable in galactic halos; on the other hand, it can be used as an estimator for coarse-grained distribution function of halo particles. Namely, for non-relativistic dark matter particles

$$Q \simeq m^4 \cdot \frac{n}{\langle \frac{1}{3} p^2 \rangle^{3/2}} ,$$

where m is the mass of these particles and n is their average number density in a halo. Assuming that the coarse-grained distribution of halo particles is isotropic, $f_{\text{halo}}(\mathbf{p}, \mathbf{r}) =$

$f_{\text{halo}}(p, r)$, one estimates

$$\frac{n}{\langle p^2 \rangle^{3/2}} = \frac{[\int f_{\text{halo}}(\mathbf{p}, \mathbf{r}) d^3\mathbf{p}]^{5/2}}{[\int f_{\text{halo}}(\mathbf{p}, \mathbf{r}) \mathbf{p}^2 d^3\mathbf{p}]^{3/2}} \sim f_{\text{halo}}(p_*, r),$$

where p_* is a typical momentum of the dark matter particles. In this way the magnitude of the coarse-grained distribution function in galactic halos is estimated as

$$f_{\text{halo}} \simeq \frac{Q}{3^{3/2} m^4}. \tag{1.3}$$

Coarse-grained distribution function is known to decrease during violent relaxation in collisionless systems [16]. Hence, the primordial phase space density of dark matter particles cannot be lower than that observed in dark halos. This leads to the Tremaine-Gunn-like constraints on dark matter models [13, 15]. The strongest among these constraints are obtained by making use of the highest phase space densities observed in dark halos, namely those of dwarf spheroidal galaxies (dSph) [11, 14]. dSph's are the most dark matter dominated compact objects, and seem to be hosted by the smallest halos containing dark matter [11]. In recently discovered objects Coma Berenices, Leo IV and Canes Venaciti II, the value of Q ranges from $5 \cdot 10^{-3} \frac{M_{\odot}/\text{pc}^3}{(\text{km/s})^3}$ to $2 \cdot 10^{-2} \frac{M_{\odot}/\text{pc}^3}{(\text{km/s})^3}$ [17]. In what follows we use the first, more conservative value,

$$Q = 5 \cdot 10^{-3} \frac{M_{\odot}/\text{pc}^3}{(\text{km/s})^3}. \tag{1.4}$$

By requiring that the primordial distribution function exceeds the coarse-grained one, $f > f_{\text{halo}}$, one arrives at the constraint

$$3^{3/2} m^4 f > Q. \tag{1.5}$$

This constraint gives rise to a reasonably well defined lower bound on m in a given model.

If the primordial distribution is such that (1.5) is barely satisfied, the formation of high- Q objects like dSph's is suppressed. In fact, it may be suppressed even for larger f , since the coarse-grained distribution function may decrease considerably during the evolution. The parameter

$$\Delta \equiv \frac{3^{3/2} m^4 f}{Q}$$

shows how strongly the coarse-grained distribution function f must be diluted due to relaxation processes in order that the formation of dense compact dark matter halos be suppressed. It is known from simulations that the phase space density decreases during the structure formation. In particular, during the nonlinear stage it decreases by a factor of 10^2 to 10^3 [18], or possibly higher. Hence, the primordial distribution function of WDM particles should be such that $\Delta \gtrsim 10^2 - 10^3$. At least naively, obtaining the dilution factor in a given model in the ballpark $\Delta = 1 - 10^3$ would indicate that the primordial phase space density is just right to make dwarf galaxies but not even more compact objects. Interestingly, we will find that Δ is indeed in this ballpark for WDM gravitinos obeying (1.2).

As discussed in ref. [19], only a fraction of dark matter particles should definitely have high phase space density obeying (1.5). This fraction ν is estimated as the relative contribution of dSph's into the total mass density of dark matter. Using the dwarf number density $n_{\text{dwarf}} \simeq 7 \cdot 10^{-2} \text{ Mpc}^{-3}$ from ref. [20] and assuming the average dSph mass of order $10^7 M_{\odot}$ [11], one estimates

$$\nu \simeq \frac{\Omega_{\text{dSph}}}{\Omega_{\text{DM}}} \simeq 10^{-5} . \tag{1.6}$$

To be on conservative side, we impose the constraint (1.5) on this fraction of WDM particles only. Also, we calculate the value of Δ for this fraction. One expects that once the right fraction of the dark matter particles has the high phase space density, the most compact objects are produced in right numbers (and not overproduced). We have checked that our results would change very little if we used an estimate for ν differing from (1.6) even by an order of magnitude, i.e. $\nu = 10^{-6} - 10^{-4}$.

To summarize, we consider a dark matter model viable if a fraction (1.6) of its particles has primordial phase space density obeying (1.5) with Q given by eq. (1.4).

In this paper we make use of this phase space density criterion together with the bound (1.2) to examine light gravitino as a warm dark matter candidate, assuming that R-parity is conserved and hence gravitino is stable. We find that gravitino mass should be in the range

$$1 \text{ keV} \lesssim m_{\tilde{G}} \lesssim 15 \text{ keV} ,$$

cf. [9].

In the early Universe, light gravitinos are produced in decays of superparticles and in scattering processes [21–24]. For so light gravitinos, their production in decays of superparticles plays an important role [25]. We consider this mechanism in section 2.1, where we also evaluate the spectrum of produced gravitinos. In section 2.2 we discuss gravitino production in scattering processes. The latter mechanism operates most efficiently at the highest possible temperatures in the early Universe, so the requirement that gravitinos are not overproduced restricts severely the reheat temperature T_{R} , cf. [25, 26]; we find that T_{R} must be at most in the TeV range.

Most notably, gravitinos serve as warm dark matter candidates only if other superparticles are rather light. We find that superparticles whose mass M is below the reheat temperature should obey

$$M \lesssim 350 \text{ GeV} , \tag{1.7}$$

otherwise gravitinos are overproduced in their decays and in scattering and/or relic gravitinos are too cold. Barring fine tuning between the reheat temperature in the Universe and superparticle masses, this means that gravitino as warm dark matter candidate will soon be either ruled out or supported by the LHC experiments.

The bound (1.7) is to be compared to the experimental bounds on masses of gluino and quarks of the 1st and 2nd generations, $M_{\tilde{q},\tilde{g}} \geq 250 - 325 \text{ GeV}$ [2]. Given the narrow interval between these bounds, we find it disfavored that squarks and gluinos participate in gravitino production processes. Hence, we elaborate also on a scenario with relatively

light colorless superparticles whose masses M obey (1.7), heavy squarks and gluinos, and reheat temperature in between,

$$M \lesssim T_R \ll M_{\tilde{q},\tilde{g}}. \tag{1.8}$$

In this scenario, squarks and gluinos do not play any role in gravitino production, while the important production processes are decays and collisions of sleptons, charginos and neutralinos. We find that in this case, the overall picture is consistent in rather wide range of parameters, with the reheat temperature extending up to 10 TeV.

It is worth noting that for light gravitino we consider in this paper, the lifetime of next-to-lightest superparticles (NLSP) is short, $\tau_{\text{NLSP}} \lesssim 2 \cdot 10^{-5}$ s. Thus, their decays after decoupling are not hazardous for BBN. On the other hand, in the mass range of gravitino and superpartners we have found favored, one has $\tau_{\text{NLSP}} \gtrsim 5 \cdot 10^{-7}$ s and thus the NLSP decay length (neglecting γ -factor) is in the range

$$160 \text{ m} \lesssim c\tau \lesssim 7 \text{ km}.$$

So, with gravitino WDM, it is likely that NLSP (if chargeless and colorless) will freely travel through the LHC detectors.

2. Gravitino production mechanisms

2.1 Production in decays

We begin with the study of the light gravitino production in two-body decays of thermalized superparticles, assuming that the reheat temperature in the Universe exceeds considerably the masses of these superparticles. Let us first find the distribution function of gravitinos produced in decays of one kind of superparticles with mass M . At time t , these superparticles have thermal distribution function $f_{\text{th}}(p, t)$.

The Boltzmann equation for the gravitino distribution function $f(p, t)$ is

$$\frac{\partial f(p, t)}{\partial t} - H(t)p \frac{\partial f(p, t)}{\partial p} = I,$$

where $H(t)$ is the Hubble parameter. In this section we consider the contribution into the collision term I that comes from two-body decays and is given by

$$I = \frac{1}{2|\mathbf{p}|} \int \frac{d^3P}{2E(2\pi)^3} \frac{d^3p'}{2|\mathbf{p}'|(2\pi)^3} (2\pi)^4 \delta^{(4)}(P - p - p') f_{\text{th}}(P, t) |\mathcal{M}|^2.$$

$E = \sqrt{M^2 + \mathbf{P}^2}$ is the energy of the decaying particle. The amplitude \mathcal{M} is related to the decay rate in the rest frame of the decaying particle as $|\mathcal{M}|^2 = 16\pi M\Gamma$. Neglecting the masses of particles in the final state, one has¹ [27]

$$\Gamma = \frac{M^5}{6m_G^2 M_{\text{Pl}}^2}.$$

¹This formula, generally speaking, does not work in the Higgs-higgsino sector, and in some region of the parameter space the corresponding decays are suppressed. We treat higgsinos on equal footing with other charginos and neutralinos in what follows. Refining this approximation would not change our results considerably.

We note that both decays and scattering processes produce longitudinal gravitino (goldstino), so the number of gravitino helicity states effectively equals two. We also note that in the parameter range of interest, the inverse $2 \rightarrow 1$ processes, leading to disappearance of gravitino, have negligible rates.

Upon integrating over the momentum of the SM particle and over the direction of \mathbf{P} , the collision term takes the following form,

$$I = \frac{M\Gamma}{p^2} \int_{E_{\min}}^{\infty} f_{\text{th}}(P, t) dE ,$$

where

$$E_{\min} = p + \frac{M^2}{4p}$$

is the minimum energy of the decaying particle capable of producing gravitino of momentum p . Of particular interest for what follows is the low momentum region, $p \ll M, T$. In that case $E_{\min} \gg M$, i.e., slow gravitinos are born in peculiar decays of fast moving superparticles, which produce gravitinos in a narrow backward cone. For this reason, the efficient production of slow gravitinos occurs at temperatures $T \gtrsim M^2/p \gg M$. As we will see shortly, for relatively low reheat temperatures T_{R} this results in a non-trivial shape of the gravitino spectrum at low momenta, with a cutoff at $p/T \sim M^2/T_{\text{R}}^2$.

It is convenient to take comoving momentum $q = a(t)p$ as the argument of the gravitino distribution function. Here $a(t)$ is the scale factor, whose present value is normalized to unity, $a(t_0) = 1$. The Boltzmann equation takes the form

$$\frac{df(q, t)}{dt} = \frac{M\Gamma}{q^2} a^2(t) \int_{E_{\min}}^{\infty} f_{\text{th}}(P, t) dE .$$

It can be easily integrated, giving

$$f(q, t) = \int_{t_{\text{R}}}^t dt' \frac{M\Gamma}{q^2} a^2(t') \int_{E_{\min}}^{\infty} f_{\text{th}}(P, t') dE ,$$

where t_{R} refers to the beginning of the thermal phase of the cosmological evolution after reheating. Hereafter we assume that the production of gravitinos is negligible at the reheating epoch. This is of course an arbitrary assumption reflecting our ignorance of the reheating mechanism; we expect that the gravitino production at reheating, if any, would make the regions of favored gravitino and superparticle masses even narrower as compared to the regions presented below.

Since the thermal distribution function $f_{\text{th}}(P, t)$ of the decaying particles depends on the ratio $E/T(t)$ only, it is convenient to trade the integration over production time for the integration over temperature. To this end we use the entropy conservation and the relation $T = \sqrt{\frac{M_{\text{Pl}}^*}{2t}}$ with $M_{\text{Pl}}^* \equiv M_{\text{Pl}} \sqrt{\frac{90}{8\pi^3 g_*}}$, valid at the radiation domination epoch. Thus, if

the gravitino distribution function had not been distorted by structure formation, at the present epoch it would have been given by

$$f(q, t_0) = \int_0^{T_R} dT \frac{M \Gamma M_{\text{Pl}}^* T_{0,\text{eff}}^2}{q^2 T^5} \int_{E_{\text{min}}}^{\infty} f_{\text{th}} \left(\frac{E}{T} \right) dE.$$

Here $T_{0,\text{eff}} \equiv T_0 \left(\frac{g_{*0}}{g_*} \right)^{1/3}$; g_* and $g_{*0} \equiv \frac{43}{11}$ are the effective number of relativistic degrees of freedom at gravitino production and at present epoch, respectively; in the framework of MSSM with all superparticles relativistic in the plasma $g_* = g_{\text{MSSM}} = 228.75$ and $T_{0,\text{eff}} \approx 0.7$ K.

Changing the variables $(T, E) \rightarrow (z = \frac{E}{T}, x = \frac{M}{T})$ and performing the integration over x we obtain finally the following result for the primordial distribution function expressed in terms of the momenta redshifted to the present epoch,

$$\begin{aligned} f(p) \equiv f(q, t_0) &= \frac{8}{3} \frac{M_{\text{Pl}}^* \Gamma}{M^2} \left(\frac{T_{0,\text{eff}}}{p} \right)^2 \cdot I \left(\frac{p}{T_{0,\text{eff}}}, \frac{M}{T_R} \right) \\ &= \frac{2\sqrt{5}}{3\pi^{3/2} \sqrt{g_*}} \frac{M^3}{m_G^2 M_{\text{Pl}}} \left(\frac{T_{0,\text{eff}}}{p} \right)^2 \cdot I \left(\frac{p}{T_{0,\text{eff}}}, \frac{M}{T_R} \right), \end{aligned} \quad (2.1)$$

where

$$I \left(\frac{p}{T_{0,\text{eff}}}, \frac{M}{T_R} \right) \equiv \int_{z_{\text{min}}}^{\infty} \left[\left(\frac{p}{T_{0,\text{eff}}} \right)^{3/2} \left(z - \frac{p}{T_{0,\text{eff}}} \right)^{3/2} - \left(\frac{M}{2T_R} \right)^3 \right] f_{\text{th}}(z) dz \quad (2.2)$$

with

$$z_{\text{min}} = \frac{p}{T_{0,\text{eff}}} + \frac{M^2 T_{0,\text{eff}}}{4 T_R^2 p}.$$

The corresponding spectrum $\frac{dn}{dp} = 4\pi p^2 f(p)$ for $T_R \gg M$ is shown in the left panel of figure 2 in comparison with the thermal spectrum at temperature $T_{0,\text{eff}}$ and the same total number of particles. It is seen that gravitinos produced in decays have lower average momentum and in this sense are cooler than thermal ones. The overall shape of the spectrum is not of particular interest for our purposes, however: the formation of compact objects like dSph's depends on the low-momentum part of the spectrum, where the phase space density $f(p)$ is high.

As we alluded to above, the low-momentum part of the spectrum depends in a peculiar way on the ratio of the mass of the decaying particle M to the reheat temperature T_R . This comes out from eq. (2.2), in particular, through the lower limit of integration z_{min} . For $T_R \rightarrow \infty$, the distribution function at low momenta, $p \ll T_{0,\text{eff}}$, is given by

$$\begin{aligned} f(p) &= \frac{2\sqrt{5}}{3\pi^{3/2} \sqrt{g_*}} \frac{M^3}{m_G^2 M_{\text{Pl}}} \left(\frac{T_{0,\text{eff}}}{p} \right)^{1/2} \int_0^{\infty} z^{3/2} f_{\text{th}}(z) dz \\ &= \frac{\sqrt{5} \zeta(5/2)}{16\pi^4 \sqrt{g_*}} \frac{M^3}{m_G^2 M_{\text{Pl}}} g^{c_{\text{dec}}} \left(\frac{T_{0,\text{eff}}}{p} \right)^{1/2}. \end{aligned}$$

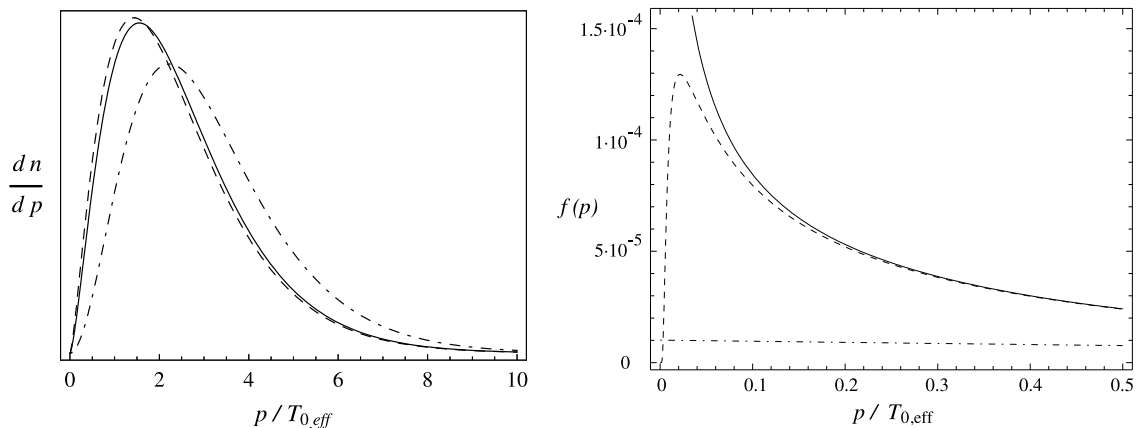


Figure 2: *Left:* Spectra of gravitinos produced in decays of thermalized fermions (solid) and bosons (dashed) in comparison with the Fermi-Dirac (dash-dotted) spectrum at temperature $T_{0,\text{eff}}$, normalized to the same total number of particles. *Right:* Low momentum part of the distribution functions of gravitinos with $m_{\tilde{G}} = 10$ keV produced in the decays of bosons with $M = 200$ GeV for $T_{\text{R}} \rightarrow \infty$ (solid line) and $T_{\text{R}} = 4M$ (dashed line). Dash-dotted line: Fermi-Dirac distribution at temperature $T_{0,\text{eff}}$, normalized to the same total number of particles.

where g is the number of helicity states of the decaying particle, and $c_{\text{dec}} = 1$ for bosons and $c_{\text{dec}} = \left(1 - \frac{1}{2\sqrt{2}}\right)$ for fermions. As expected, the combination $m_{\tilde{G}}^4 f$ entering (1.5) increases with the gravitino mass, $m_{\tilde{G}}^4 f \propto m_{\tilde{G}}^2$, so that only light gravitinos are warm. We note in passing that the distribution function derived in this way is unbounded as $p \rightarrow 0$. However, the correct distribution function of gravitinos — fermions with effectively two helicity states — cannot exceed the value $2/(2\pi)^3$ because of Pauli-blocking. To take this into account we simply cut the distribution function at $2/(2\pi)^3$ wherever the calculated distribution function exceeds this value. In fact, this procedure is used almost nowhere in the parameter space we consider in this paper, as the calculated distribution function almost never exceeds $2/(2\pi)^3$.

For finite T_{R} , but still $T_{\text{R}} \gtrsim M$, the distribution function (2.1) no longer peaks at $p \rightarrow 0$. Instead, it has a rather broad peak at $p/T \sim M^2/T_{\text{R}}^2$ and exponentially decays towards $p \rightarrow 0$. This is shown in the right panel of figure 2, where we also compare the distribution function of gravitinos produced in decays with the thermal distribution function at temperature $T_{0,\text{eff}}$ normalized to the same total number of produced particles. We again see that the gravitinos produced in decays are substantially cooler than fermions with thermal distribution, as the maximum phase space density is substantially higher in the former case.

It is clear from (2.1) that the largest contribution into the gravitino production comes from the heaviest superparticles that have ever been relativistic in cosmic plasma. To get an idea of numerics, let us consider the case in which g_b bosonic and g_f fermionic superparticle degrees of freedom have one and the same mass M , and the reheat temperature is substantially higher than M . Then the present number density of gravitinos produced in

decays of these superparticles is

$$n_0^{\text{dec}} = \int f(p) d^3p = \frac{3\sqrt{5}\zeta(5)}{16\pi^{5/2}\sqrt{g_*}} T_{0,\text{eff}}^3 \frac{M^3}{m_{\tilde{G}}^2 M_{\text{Pl}}} \left(g_b + \frac{15}{16} g_f \right),$$

and the present mass density of these gravitinos is given by

$$\Omega_{\tilde{G}}^{\text{dec}} = \frac{m_{\tilde{G}} n_0^{\text{dec}}}{\rho_c} \approx 8 \cdot 10^{-4} \left(g_b + \frac{15}{16} g_f \right) \left(\frac{g_{\text{MSSM}}}{g_*} \right)^{3/2} \left(\frac{1 \text{ keV}}{m_{\tilde{G}}} \right) \left(\frac{M}{100 \text{ GeV}} \right)^3. \quad (2.3)$$

A crude estimate for the gravitino mass is obtained by assuming that the distribution function of $\nu = 10^{-5}$ of gravitinos is roughly comparable to the Pauli-blocking value, $f = 2/(2\pi)^3$. Then the condition (1.5) corresponds to

$$m_{\tilde{G}} > 1 \text{ keV}. \quad (2.4)$$

As an example, if the heaviest superparticles are squarks of the 1st and 2nd generations and gluinos, as motivated by mSUGRA, if they have the same mass and the reheat temperature is high enough so that these particles were relativistic in the cosmic plasma, then $g_b = g_{\tilde{q}} = 4 \cdot 3 \cdot 4 = 48$, $g_f = g_{\tilde{g}} = 2 \cdot 8 = 16$ and $g_* = g_{\text{MSSM}}$. Making use of the estimate (2.3) and the upper limit on warm gravitino mass (1.2), we find in this example that the common mass of squarks and gluinos must be rather small, $M_{\tilde{q},\tilde{g}} \lesssim 350 \text{ GeV}$, otherwise gravitinos are overproduced. We will refine these estimates in section 3.

2.2 Gravitino production in scattering

Gravitino production in scattering processes has been worked out in refs. [22, 23] using the Braaten-Yuan prescription and hard loop resummation. It has been reconsidered recently in ref. [24] with the results substantially different from those of refs. [22, 23] in some regions of parameter space. We will use the approach of refs. [22, 23] with understanding that there is considerable uncertainty both in gravitino production rate and in their spectrum, especially at relatively low temperatures, $T \sim M$. We will further comment on this uncertainty in section 3.

The contribution of scattering into the gravitino production is dominated by the processes involving the heaviest superparticles which have ever been relativistic in the cosmic plasma. Furthermore, this contribution strongly depends on whether or not these superparticles are colored. In what follows we consider two scenarios which we think are representative for realistic supersymmetric extensions of the Standard Model. Our analysis below is straightforwardly redone for the general case, but given the unknown superparticle spectrum and the uncertainty in (1.5), considering these simple scenarios will be sufficient for our purposes. The first scenario has been described in the end of section 2.1: in this scenario the heaviest are squarks of the 1st and 2nd generations and gluinos, and we assume that they all have the same mass M and that the reheat temperature exceeds M . Given the experimental bounds, $M \geq 250 - 325 \text{ GeV}$, it is clear already from the preliminary discussion in the end of section 2.1 that this scenario may be consistent only in a rather narrow range of the parameter space. Hence, we discuss also the second scenario, which is defined

by the relation (1.8) where M is the common mass of sleptons, charginos and neutralinos. In the second scenario, squarks and gluinos play no role in the gravitino production in the early Universe. Given the strong dependence on the mass M , varying the rest of SUSY parameters in either scenario does not lead to significant changes of our results.

For the first, squark-gluino scenario, the results of ref. [23] apply directly, so the mass density of gravitinos produced in scattering is given by

$$\Omega_{\tilde{G}}^{\text{sc}} \approx \omega_s g_s^2 \ln\left(\frac{k_s}{g_s}\right) \left(\frac{M}{100 \text{ GeV}}\right)^2 \left(\frac{1 \text{ keV}}{m_{\tilde{G}}}\right) \left(\frac{T_R}{1 \text{ TeV}}\right), \quad (2.5)$$

where g_s is the strong coupling constant at the energy scale T_R , and $\omega_s = 0.732$, $k_s = 1.271$.

For the second, color-singlet scenario, the results of ref. [23] have to be modified. To this end, we consider electroweak scattering processes only and omit the contributions of reactions with external squarks. Also, we omit the squark contributions into the thermal masses of the gauge bosons. The overall gravitino production cross section depends on thermal masses m_{th} as $\ln(T/m_{\text{th}})$ and thus grows as the thermal mass decreases. As a result, the gravitino production cross section in our scenario is nearly 80% of the electroweak part obtained in ref. [23], although 1/3 of all processes are omitted. Using the modified cross sections and $g_* = 142.75$, we find for the present mass density of gravitinos produced in $2 \rightarrow 2$ processes in primordial plasma:

$$\Omega_{\tilde{G}}^{\text{sc}} \approx \sum_{\alpha=1}^2 \omega_{\alpha} g_{\alpha}^2 \ln\left(\frac{k_{\alpha}}{g_{\alpha}}\right) \left(\frac{M}{100 \text{ GeV}}\right)^2 \left(\frac{1 \text{ keV}}{m_{\tilde{G}}}\right) \left(\frac{T_R}{1 \text{ TeV}}\right), \quad (2.6)$$

with modified constant factors $\omega_{\alpha} \approx (0.152, 0.372)$ and scales in logarithms $k_{\alpha} \approx (1.52, 1.52)$. Here $\alpha = 1$ and $\alpha = 2$ refer to the gauge groups $U(1)_Y$ and $SU(2)_L$, respectively, with the gauge couplings $g_{\alpha} = (g', g)$.

The estimates (2.5) and (2.6) have considerable uncertainties related to infrared problems existing in field theory at finite temperature. These will translate into uncertainties in our estimates presented in section 3.

3. Results

There are three criteria the WDM model with light gravitino should satisfy. First, as discussed in Introduction, gravitino would serve as *warm* dark matter provided its mass satisfies the upper bound (1.2), $m_{\tilde{G}} \lesssim 15 \text{ keV}$. Another criterion is that the present gravitino mass density should be equal to the observed dark matter density. In both scenarios of section 2.2, the total gravitino mass density is the sum of contributions due to the decay and scattering processes, so that one requires $\Omega_{\tilde{G}}^{\text{dec}} + \Omega_{\tilde{G}}^{\text{sc}} = \Omega_{\text{DM}} \approx 0.2$. This requirement gives one relation between the three parameters, the masses $m_{\tilde{G}}$, M and reheat temperature T_R in each scenario. For the first, squark-gluino scenario we make use of (2.5) as well as (2.3) with $g_b = 48$, $g_f = 16$ and $g_* = g_{\text{MSSM}}$. For the second, color-singlet scenario the appropriate expressions are (2.6) and (2.3) with $g_b = g_l = 3 \cdot (4 + 2) = 18$, $g_f = g_{\tilde{\chi}} = 4 \cdot 2 + 2 \cdot 4 = 16$ and $g_* = 142.75$. In either case, scanning the reheat temperature

from $T_R \sim M$ upwards, we observe from eqs. (2.3), (2.5) and (2.6) that this criterion gives a lower bound on the gravitino mass for given M .

The third criterion is discussed in section 1: about 10^{-5} of gravitinos should have the primordial phase space density obeying (1.5) with $m \equiv m_{\tilde{G}}$. This criterion gives a lower bound on the gravitino mass for given M and T_R . This bound has to do with the magnitude of the gravitino distribution function at low momenta where this function is large. Instead of calculating the low momentum part of the distribution function of gravitinos produced in the scattering processes, we first use the lower bound on the overall distribution function, which is obtained by neglecting altogether the contribution of the scattering processes into the distribution function in the low momentum region. The lower bounds on $m_{\tilde{G}}$ obtained within this decay dominance approximation are overestimated in comparison with those one would obtain by the complete treatment. To get an idea of the uncertainty introduced by approximating the low momentum part of the distribution function by the contribution of the decay processes only, we then add the contribution from scattering assuming that gravitinos produced in the latter way have thermal-shaped distribution (1.1), but normalized to the total mass density, eqs. (2.5) and (2.6) in the first and second scenario, respectively,

$$f^{\text{sc}}(p) = \frac{\rho_c \Omega_{\tilde{G}}^{\text{sc}}}{6\pi\zeta(3)m_{\tilde{G}} T_{0,\text{eff}}^3} \frac{1}{e^{p/T_{0,\text{eff}}} + 1}. \quad (3.1)$$

Within either approximation, for each set of parameters we find the value f of the phase space density, such that 10^{-5} of gravitinos have the distribution function exceeding f , and require that f obeys (1.5) at an allowed point in the parameter space.

The resulting bounds in $(M, m_{\tilde{G}})$ plane are shown in figure 3 and in figure 4 for the first and second scenario, respectively. Contours of equal T_R/M are plotted with dashed-dotted lines. At the same time, these contours correspond to constant fractions of gravitinos produced in scattering and decay channels, providing together the correct present mass density of dark matter, $\Omega_{\tilde{G}} = \Omega_{\text{DM}}$. The labels show the fraction of gravitino produced in decays. The shaded regions are allowed by both (1.2) and (1.5). We also show the lines of equal dilution factor Δ ; the dashed lines correspond to the decay dominance approximation, while the solid lines are obtained under the assumption that the scattering contribution to the low momentum part of the distribution function has the form (3.1).

In view of substantial uncertainty in the production of gravitinos in scattering, the estimates for the reheat temperature should be considered as indicative only. This is particularly relevant for the upper left parts of figures 3 and 4, where production in scattering dominates over production in decays. Also, the estimates for the dilution factor Δ are uncertain in these parts of the parameter space, due to the large uncertainty in the low momentum part of the spectrum of gravitinos produced in scattering. This is reflected by the fact that dashed and solid lines deviate significantly from each other in the upper left parts of figures 3 and 4. Furthermore, even though the thermal-shaped distribution (3.1) is a plausible approximation, we cannot exclude the possibility that scattering contribution to the distribution function of gravitinos is much larger at low momenta as compared to (3.1). In the latter case the lines of equal dilution factor Δ would shift even further

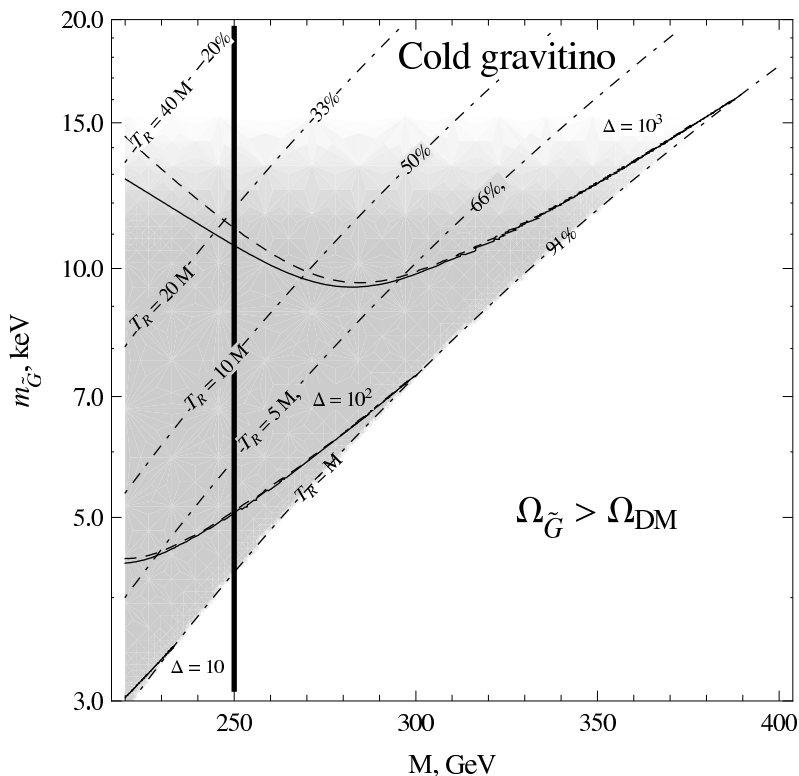


Figure 3: Allowed region of masses (shaded) in a scenario with heavy gluinos and quarks of the 1st and 2nd generations, $M_{\tilde{g}} = M_{\tilde{q}} = M$, and $T_R \gtrsim M$. Contours of equal dilution factor Δ are also shown (solid and dashed lines). The dashed lines correspond to the decay dominance approximation, while the solid lines are obtained under the assumption that the scattering contribution to the low momentum part of the distribution function has the form (3.1). Contours of equal T_R/M are shown with dash-dotted lines, on which the fraction of gravitinos produced in decays is also indicated. Conservative experimental lower bound on masses of gluinos and squarks of the 1st and 2nd generations is indicated by solid vertical line.

down. In any case, the most conservative lower bound on the gravitino mass independent of the distribution function is given by (2.4).

Irrespectively of these uncertainties, we see that in both scenarios, the relevant superparticle masses must be rather low, $M < 320 - 350$ GeV, provided that the reheat temperature is $T_R \gtrsim M$. Extending the mass range of superparticles towards larger M in either scenario would require increasingly strong fine tuning between the reheat temperature and these masses. This fine tuning is needed to ensure that superparticles are non-relativistic and hence not so numerous, but have just right abundance at the beginning of the thermal stage of the cosmological evolution to produce just right number of gravitinos. We consider this possibility implausible.

Figure 5 shows the same bounds as in figure 4 in $(T_R, m_{\tilde{G}})$ plane. On dash-dotted lines the total density of gravitinos produced in both channels is equal to the observed dark matter density for indicated superpartner masses M .

We conclude that unlike in the WIMP case, gravitino WDM does not automatically

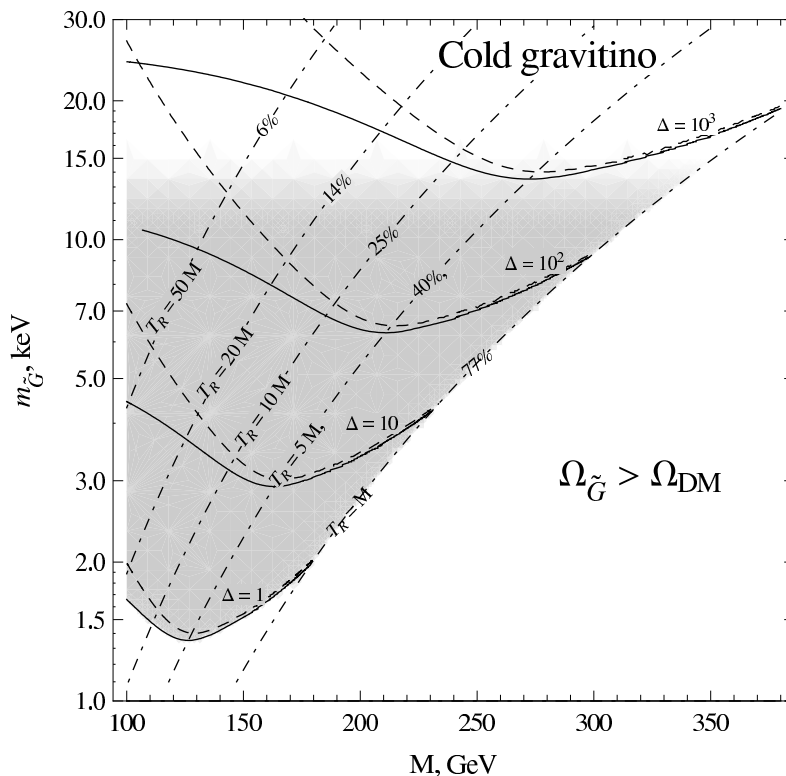


Figure 4: Same as in figure 3, but for a scenario with color singlet superparticles of equal mass M , heavy squarks and gluinos, and intermediate reheat temperature, $M \lesssim T_R \ll M_{\tilde{q}, \tilde{g}}$.

have the present mass density in the right ballpark. If the heaviest superparticles are squarks and gluinos, and they were relativistic in the cosmic plasma (the first scenario), the allowed range of parameters is rather narrow, as seen from figure 3. We consider least contrived the possibility that the masses of sleptons, charginos and neutralinos are in the range $M = 150 - 300$ GeV, the reheat temperature is $T_R = 200$ GeV – 10 TeV and the masses of gluinos and squarks are higher, $M_{\tilde{g}, \tilde{q}} \gg T_R$ (second scenario). Then for masses $m_{\tilde{G}} = 1 - 15$ keV, gravitinos can indeed serve as warm dark matter particles. In any case, gravitino as warm dark matter candidate will be either ruled out or supported by the LHC experiments.

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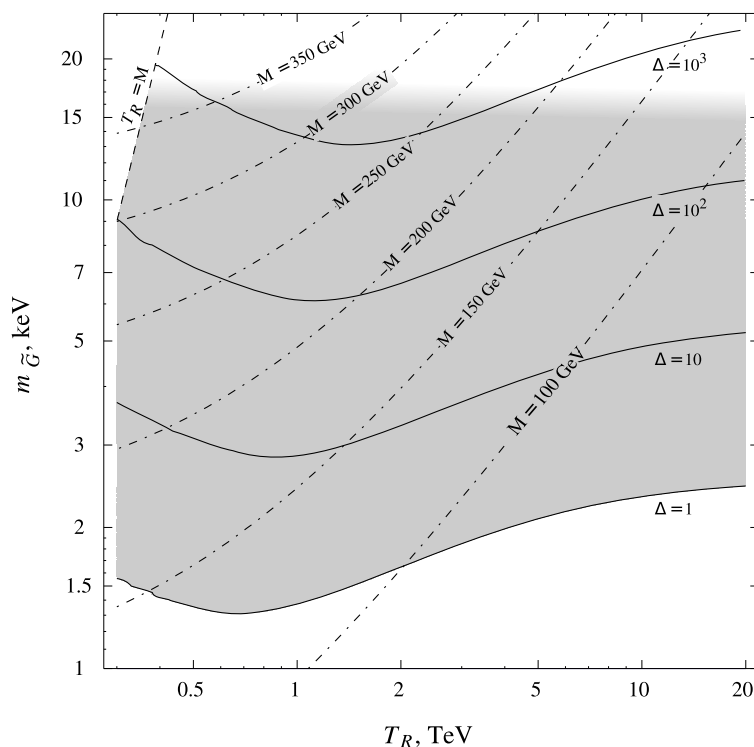


Figure 5: Region with gravitino WDM in the $(T_R, m_{\tilde{G}})$ plane for the same scenario as in figure 4. Contours of equal M are shown with dash-dotted lines.

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